



### Lab Assignment #1

## Mean Value of Periodic Time Functions

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### Objective

Measure the mean value of different rectified AC voltages, using the following input voltages: harmonic AC voltage, harmonic AC voltage with offset, and AC voltages with different slopes.

### Equipment

- Agilent 54600A Oscilloscope
- Agilent 34401A Multimeter
- Agilent 33120A Function Generator
- Agilent VEE software

### A. Theoretical Introduction

When performing AC measurements in stationary state networks, it is often not necessary to measure the time function of currents and voltages but only the peak values. Most instruments do not measure directly the peak value, but use a rectifier and a low pass filter and, therefore, a mean value proportional to the peak value is measured and displayed. In this case fatal errors can be produced if the signal is non-harmonic or if an offset is superposed. Let us analyse some special important cases.

#### 1. The General Case

The mean value of a periodic function is defined by the following equation:

$$\bar{F} = \frac{1}{T} \int_{t_0}^{t_0+T} F(t) \cdot dt,$$

where T is the period and  $t_0$  is an arbitrary the start time. Looking to a graphic display of the function F(t) we can see:

- If the areas above and below the time axis are equal, then the mean value is equal to zero;
- If an unipolar signal is used, a mean value is nonzero;
- If we look at the output signal after rectification, then the mean value is different from zero;
- The mean value is different using either a half-wave or a full-wave rectifier, furthermore, it depends on the shape of the function F(t).
- In all cases the mean value will be influenced by an offset superposed to the original signal (see the example in 2.).

If we consider the power in an AC network the RMS  $\tilde{F}$  is defined as:

$$\tilde{F} = \sqrt{\frac{1}{T} \int_{t_0}^{t_0+T} F^2(t) \cdot dt},$$



where the expression under the root is the mean squared value. Also the RMS depends on the shape of the time function. This must be taken into account if instruments displaying the RMS value are used.

All these mean values are proportional to the peak value.

## 2. Harmonic AC Voltage with Offset - Half-wave Rectifier:

### Calculation of the Mean Value

A half-wave rectifier transfers only the part  $U(t) \geq 0$  of the input voltage. The function crosses the time axis in the points  $-t^* = -\frac{T}{2\pi} \arcsin \frac{U_{\text{off}}}{\hat{U}}$  and  $\frac{T}{2} + t^*$ , respectively, with  $t^* \geq 0$  for  $U_{\text{off}} \geq 0$  (Fig. a) and  $t^* \leq 0$  for  $U_{\text{off}} \leq 0$  (Fig. b). The calculation of the mean value considers only the positive part between the two zero points. Therefore, we obtain:

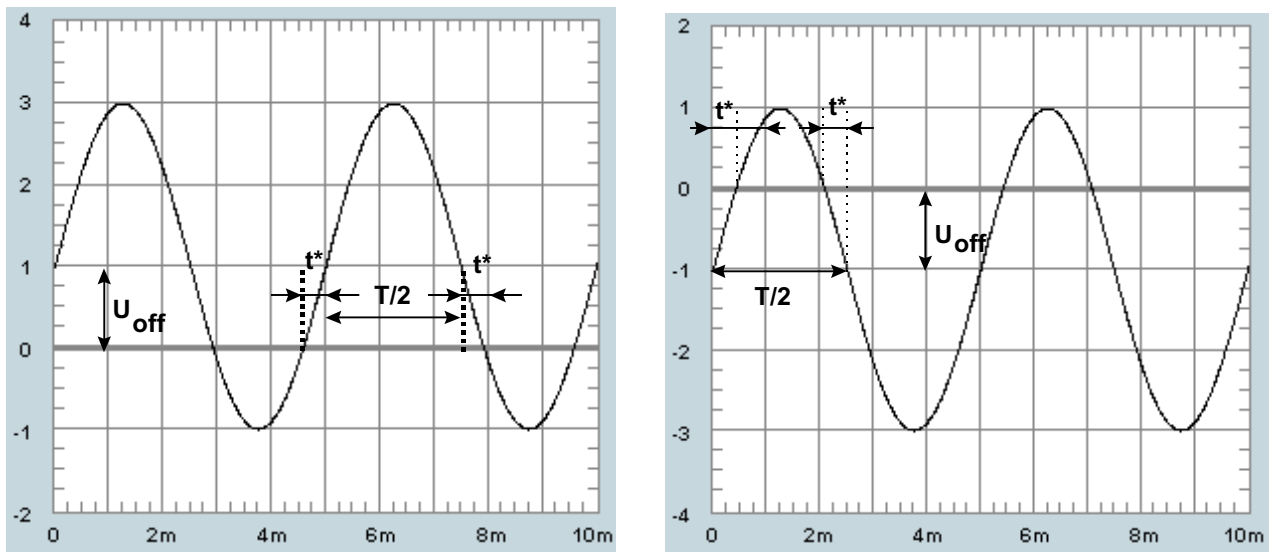


Fig.1: Harmonic Signal with Offset

a)  $U(t) = \hat{U} \sin 2\pi \frac{t}{T} + U_{\text{off}}$  with  $U_{\text{off}} \geq 0$

b)  $U(t) = \hat{U} \sin 2\pi \frac{t}{T} + U_{\text{off}}$  with  $U_{\text{off}} \leq 0$

$$\begin{aligned} \bar{U} &= \frac{1}{T} \int_{-t^*}^{\frac{T}{2}+t^*} \left( \hat{U} \sin 2\pi \frac{t}{T} + U_{\text{off}} \right) dt \\ &= \frac{1}{T} \int_{-t^*}^0 \left( \hat{U} \sin 2\pi \frac{t}{T} + U_{\text{off}} \right) dt + \frac{1}{T} \int_0^{\frac{T}{2}} \left( \hat{U} \sin 2\pi \frac{t}{T} + U_{\text{off}} \right) dt + \frac{1}{T} \int_{\frac{T}{2}}^{\frac{T}{2}+t^*} \left( \hat{U} \sin 2\pi \frac{t}{T} + U_{\text{off}} \right) dt \end{aligned}$$

Using

$$\frac{1}{T} \int_{-t^*}^0 \left( \hat{U} \sin 2\pi \frac{t}{T} + U_{\text{off}} \right) dt = \frac{1}{T} \int_0^{t^*} \left( -\hat{U} \sin 2\pi \frac{t}{T} \right) dt + \frac{1}{T} \int_0^{t^*} U_{\text{off}} dt ,$$



$$\frac{1}{T} \int_{\frac{T}{2}}^{\frac{T}{2}+t^*} \left( \hat{U} \sin 2\pi \frac{t}{T} + U_{\text{off}} \right) dt = \frac{1}{T} \int_0^{t^*} \left( -\hat{U} \sin 2\pi \frac{t}{T} \right) dt + \frac{1}{T} \int_0^{t^*} U_{\text{off}} dt ,$$

which sum to

$$\frac{2}{T} \int_0^{t^*} \left( U_{\text{off}} - \hat{U} \sin 2\pi \frac{t}{T} \right) dt ,$$

we obtain

$$\begin{aligned} \bar{U} &= \frac{1}{T} \int_0^{\frac{T}{2}} \left( \hat{U} \sin 2\pi \frac{t}{T} + U_{\text{off}} \right) dt + \frac{2}{T} \int_0^{t^*} \left( U_{\text{off}} - \hat{U} \sin 2\pi \frac{t}{T} \right) dt \\ &= \frac{1}{T} \left[ -\frac{\hat{U} \cdot T}{2\pi} \cos 2\pi \frac{t}{T} + U_{\text{off}} \cdot t \right]_0^{\frac{T}{2}} + \frac{2}{T} \left[ \frac{\hat{U} \cdot T}{2\pi} \cos 2\pi \frac{t}{T} + U_{\text{off}} \cdot t \right]_0^{t^*} \\ &= \frac{\hat{U}}{\pi} + \frac{U_{\text{off}}}{2} + \frac{\hat{U}}{\pi} \left[ \cos \left( \arcsin \frac{U_{\text{off}}}{\hat{U}} \right) - 1 \right] + \frac{U_{\text{off}}}{\pi} \arcsin \frac{U_{\text{off}}}{\hat{U}} \end{aligned}$$

and, finally,

$$\Rightarrow \bar{U} = \frac{\hat{U}}{\pi} \cos \left( \arcsin \frac{U_{\text{off}}}{\hat{U}} \right) + U_{\text{off}} \left( \frac{1}{2} + \frac{1}{\pi} \arcsin \frac{U_{\text{off}}}{\hat{U}} \right) .$$

## B. Simulation Experiments

For better understanding of the discussion use the simulation **MEAN.VXE**. To run this program you need the run-time version of Agilent VEE (Version 5 or above). This program calculates the mean, mean squared and RMS value for different time functions. You can investigate the influence of shape and offset directly as well as after rectification.

An interesting exercise is the comparison of calculated values with experimental results (see the following laboratory experiment).

## C. Laboratory Experiments

### 1. Exercises

1.1. Measure the mean value of different rectified AC voltages. Use the following input voltages:

- harmonic AC voltage,
- harmonic AC voltage with offset,
- AC voltages with different slopes.



### 2. Theoretical Background

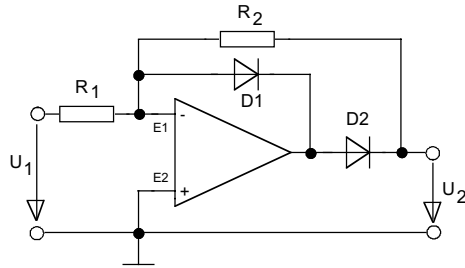


Fig.2: Precision Half-wave Rectifier

Precision rectifiers have a linear forward characteristic without knee voltage,  $U_s$ . This is the reason for their application in measurement technology. A simple half-wave rectifier can be realised with an operational amplifier and two diodes used in a non-linear feedback (see Fig.2). The forward characteristic is described by

$$U_2 = -\frac{R_2}{R_1} \text{ for } U_1 < 0 ,$$
$$U_2 = 0 \text{ for } U_1 > 0 .$$

The reason for this special behaviour is, that for an input voltage  $-U_s < U_1 < +U_s$  the OPamp works without feedback with the open-loop gain. Using this principle also a full-wave rectifier can be developed (see Fig.4).

A low pass filter is characterised by the cut-off frequency,  $\omega_c$ : only signals with a lower frequency are transmitted. Using a periodic signal with a period

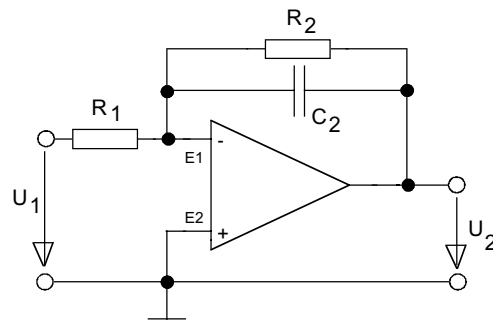


Fig.3: Low Pass Filter

$T \ll \tau = 1/\omega_c$   
( $\tau$  time constant), the output voltage is proportional to the mean value. Fig.3 shows a simple circuit of a so-called active low pass (first order filter) using an operational amplifier. The transfer function of this circuit is given by

$$\bar{U}_2 = -\frac{R_2}{R_1} \cdot \frac{1}{T} \int_0^T U_1 dt .$$

### 3. Instructions on the Laboratory Exercises

**Caution:** All circuits should be constructed using the *hps* Analog Development Board. Check conscientiously the circuit you build and determine its correct function by measurement and comparison of the signals at different circuit points using the Agilent oscilloscope. Before applying power to the circuit, please, ask the teacher to check the circuit for you.

Use the full-wave rectifier circuit shown in Fig.4 completed with a low pass shown in Fig.3 (use  $R_1 = R_2 = 10k\Omega$ ,  $C = 1\mu F$ , OPamp 5). Use the following input signals generated with the function generator Agilent 33120A:

1. Harmonic AC voltage ( $f \approx 250\text{Hz}$ ).
2. Harmonic AC voltage with offset ( $U_{\text{off}} = \pm 0.5\hat{U}$  and  $\pm\hat{U}$ ).
3. AC voltages with different other slopes.

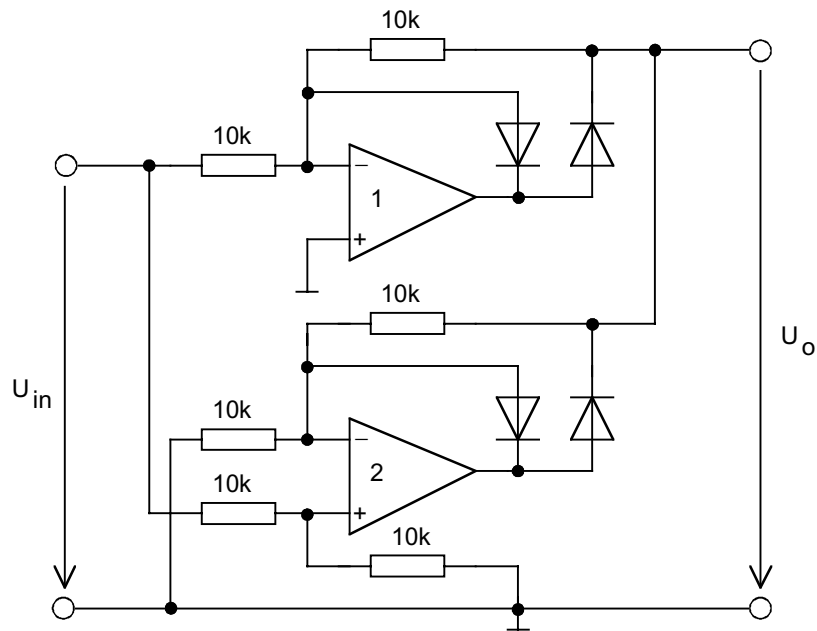


Fig.4: Full-wave Rectifier

Measure all input signals using the oscilloscope Agilent 54600 and calculate the mean value from their analytic formula.  
Measure the output voltage using the multimeter Agilent 34401A and compare the calculated and measured results.